EFFECT OF ROTATION OF THE INNER CYLINDER ON THE EQUILIBRIUM CONDITION OF SPHERICAL PARTICLES IN A TURBULENT ASCENDING FLOW IN AN ANNULAR CHANNEL

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The equilibrium condition of a solid spherical particle in a turbulent flow is considered. A relation between the rotational velocity of the inner cylinder and the critical velocity of the ascending flow in an annular channel is obtained for the range of Reynolds numbers $10^3 - 5 \cdot 10^4$.

When the inner cylinder rotates, the motion of an ascending flow in an annular channel is no longer translational, but helical with the helix angle depending on the rate of rotation of the inner cylinder. Figure 1 shows photographs of the structure of the turbulent flow in such a channel.

In order to visualize the flow structure of pure water we used a mixture of chlorobenzene, refined mineral oil, and white mineral pigment.

It is known [1–7] that at a mean axial flow velocity $v_z = 0$ rotation of the inner cylinder of an annular channel with $r_1/r_2 = 0.4-0.95$, where r_1 and r_2 are the radii of the inside and outside cylinders, respectively, creates a tangential velocity profile with a well-expressed flow core moving at the almost constant velocity v_φ and occupying up to 80% of the channel width. In the turbulent regime v_φ is about 0.4 times the tangential velocity at the surface of the inner cylinder v_{φ_1}.

When axial motion is superimposed on this flow, the tangential component v_φ of the mean resultant velocity v_ε decreases with increase in the ratio v_z/v_{φ_1} and may reach a value of the order of $0.2v_{\varphi_1}$ [2, 3]. This is due to the interaction of the two flows and the formation of a new flow with more intense turbulence. In view of the nonlinearity of the equations describing such processes, the flow interaction does not reduce to the simple addition of intensities of turbulence. The axial flow acts on the rotational motion of the liquid and deforms the tangential velocity profile, making it fuller, which leads to an increase in the frictional force at the surface of the inner cylinder.

This reasoning is fully applicable only until Taylor eddies are formed in the gap. Taylor eddies occur in the annular gap at $v_{\varphi_1}/v_z > 8-10$ and relative gap widths r_1/r_2 of the order of 0.4–0.8 [1].

As our experiments at $v_{\varphi_1}/v_z < 20$ have shown, the presence of Taylor eddies reduces v_φ by a further 10–15%.

In order to study the equilibrium conditions of spherical particles in an annular channel with rotation of the inner cylinder we performed experiments on an apparatus consisting of two vertical, coaxial, hydraulically smooth cylinders 1300 mm long, of which the inner aluminum cylinder rotated while the outer transparent plastic cylinder was fixed.

We investigated the case of a Newtonian liquid flowing over spheres of diameter d = 4 mm made of plastic (ρ_1 = 1.61 g/cm^3) and aluminum (ρ_1 = 2.6 g/cm^3). The spheres were placed in the gap between the cylinders before the experiments began.

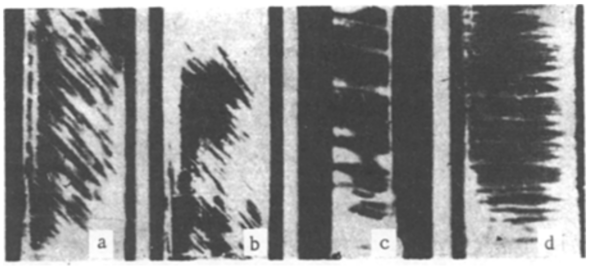

Fig. 1. Structure of single-phase turbulent flow in an annular channel with rotating inner cylinder: a) helical turbulent flow, $Re_z = 6.7 \cdot 10^3$, $Re_\varphi = 10^4$; b) the same, $Re_z = 6 \cdot 10^3$, $Re_\varphi = 10^4$; c) transition from helical flow to flow with Taylor eddies, $Re_z = 1.4 \cdot 10^4$, $Re_\varphi = 8 \cdot 10^4$; d) axially moving Taylor eddies, $Re_z = 3 \cdot 10^2$, $Re_\varphi = 10^4$.

The Reynolds number of the spiral flow varied in the range 10^3-10^4. The experiments were performed as follows. The rate of rotation of the inner cylinder v_{φ_1} was fixed. By gradually increasing the flow of water we created a steady lift force just sufficient to suspend the selected particles in the flow. The critical velocity was assumed to have been reached when the particles occupied the space between two marks 0.05 m apart in the middle of the experimental section. The position of the particles in the channel was determined visually.

It should be noted that at $v_{\varphi_1} = 0$ the particles executed a random oscillatory motion between the marks, while at $v_{\varphi_1} > 0$ they also moved in the horizontal plane along a circle close to the walls of the outer cylinder with a certain mean velocity $v_{\varphi a} = 2\pi r/t$, where $r \approx r_2 - d/2$ is the radius of curvature of the particle trajectory, and t the time required for it to make one revolution in the horizontal plane. The value of $v_{\varphi a}$ was determined experimentally.

The experimental data show that rotation of the inner cylinder has an important influence on the lift force exerted by the ascending flow. By increasing the rate of rotation it is possible to suspend the particles at a mean axial flow velocity v_z much lower than the velocity determined from the equation for the particle equilibrium condition:

$$v_z = \sqrt{G_0/C_w \rho_2 d^2},$$

where G_0 is the weight of the particle in a medium of density ρ_2, and C_W is the drag coefficient in the turbulent regime.

A solid particle in an annular channel is acted upon by: the weight of the particle in the given medium $G_0 = \frac{\pi d^3}{6}(\rho_1-\rho_2)g$ (spheres); the centrifugal force directed at any given moment in a direction normal to the particle trajectory; a force due to the radial pressure gradient in the gap; the inertia force $F_1 = dv_{\varepsilon_0}/dt$; the Coriolis force; and the drag due to flow past the particles at the resultant velocity v_{ε_0}, for a sphere $P_\varepsilon = c_w \rho_2 d^2 v^2_{\varepsilon_0} = kv^2_{\varepsilon_0}$. Since the radial pressure gradient is small [7–9], the centrifugal force $F_2 = \frac{\pi}{6} d^3(\rho_1-\rho_2)\frac{v^2_{\varphi_a}}{r}$ usually prevails.

The direction of this force is nearly radial. Under the influence of this complex three-dimensional system of forces the motion of a particle in the turbulent flow is random: it now approaches, now moves away from the wall at certain moments, experiencing the decelerating action of the wall and the layers of liquid near the wall.

This type of interaction is observed in cyclones.

Isolating the most important factors determining the entrainment of particles in the axial direction, we will average the motion of the particles in time and neglect radial displacements and the forces acting in that direction, i.e., we will consider the forces and velocities acting in a plane tangential to a cylindrical surface of some radius r characterizing the mean position of the particle in the annular space. We also neglect random fluctuations of the particle velocity, i.e., we assume that $dv/dt = 0$. We select a Cartesian coordinate system with origin at the center of the particle and consider its equilibrium conditions in averaged motion along a helical line (Fig. 2a).

For $\rho_1 > \rho_2$ the force G_0 is directed vertically downward. Averaging the effect of the decelerating impulses we can estimate them by means of a certain constant force P_φ which in the general case has vertical and tangential components. When G_0 and the lift force become equal, the particle moves through the annular space in the horizontal plane, and for this case the vertical component of the decelerating forces may be taken equal to zero.

The velocity vector diagram for this case is shown in Fig. 2b. The translational flow velocity is equal to the velocity of the flow core in the annular space $V_\varepsilon = V_z + V_\varphi$.

Above it was shown that the absolute velocity of a particle in the flow V_{φ_a} can be determined by observing its motion in the annular channel under equilibrium conditions. The particle velocity relative to the flow V_{φ_0} is equal in magnitude and opposite in direcion to the vector difference of V_ε and V_{φ_a}. The vertical component V_{z_0} of the velocity V_{ε_0} for equilibrium conditions is opposite to V_z, and the tangential component $V_{\varphi_0} = V_\varphi - V_{\varphi_a}$.

It is clear from the diagram that for any given value of V_z it is possible, by increasing V_φ (and hence V_{ε_0}), to increase the lift force exerted by the flow in the vertical direction. At a given tangential velocity the lift force reaches its maximum value when the particle is completely decelerated, i.e., when $V_\varphi = V_{\varphi_0}$, and its minimum value when $V_{\varphi_0} = 0$.

In [10] it was erroneously proposed to determine the lift force for a flat plate in the annular gap of a drilling with rotating drill pipes not from the relative tangential velocity of the particle, but from the peripheral velocity of the surface of the drill pipes. Moreover, the special characteristics of the motion of a plate in a helical flow noted later in [11] were not taken into account.

Our experiments at $v_z/v_{\varphi_1} = 0.04$–0.4 gave $v_\varphi = (0.35$–$0.4)v_{\varphi_1}$, and $v_{\varphi_a} \approx 0.5v_\varphi \approx 0.2v_{\varphi_1}$.

Thus, $v_{\varphi_0} = 0.2v_{\varphi_1}$ or in general form $v_{\varphi_0} = nv_{\varphi_1}$. It may be assumed that the value of the coefficient n depends on the shape of the particles, their size, the ratio of their size to the width of the gap δ, the particle concentration, and the conditions of interaction of v_{φ_1} and v_z.

For the equilibrium state of the particle we write

$$R_\varepsilon = G_0 = kv^2_{\varepsilon_0}\sin\alpha = k(v_z^2 + v^2_{\varphi_0})\sin\alpha. \qquad (1)$$

On the other hand, equilibrium is also reached in the absence of rotation of the liquid, by increasing the axial velocity v_z. At the critical velocity v_* the same particle is acted upon by the force

$$R_\varepsilon = kv_*^2. \qquad (2)$$

Equating (1) and (2), after transformations and allowing for the fact that

$$\sin\alpha = \frac{v_z}{\sqrt{v_z^2 + v^2_{\varphi_0}}},$$

we obtain

$$\frac{v_{\varphi_1}}{v_z} = \frac{1}{n}\sqrt{\left(\frac{v_*}{v_z}\right)^4 - 1}, \qquad (3)$$

where v_{φ_1} is the tangential velocity at the surface of the inner cylinder at which particles characterized by given values of v_* and n will be in equilibrium at $v_z < v_*$.

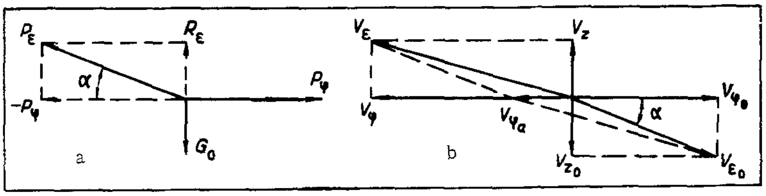

Fig. 2. Diagram of forces (a) and velocities (b) for a particle in an ascending flow through an annular channel.

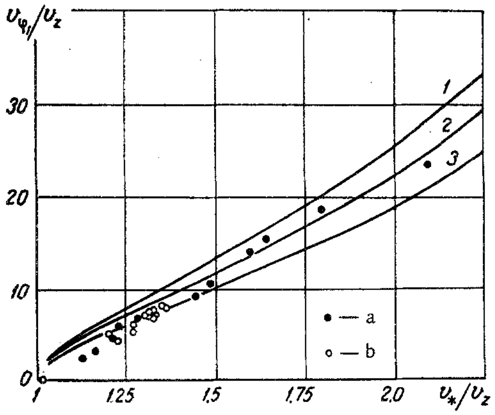

Fig. 3. Equilibrium of particle in an annular channel as a function of v_{φ_1}/v_z: 1) calculated from Eq. (3) for n = 0.15; 2) the same, 0.17; 3) the same, 0.20; a, b) spheres 4 mm in diam. with ρ_1 equal to 1.61 and 2.6 g/cm^3, respectively.

The graph (see Fig. 3) of v_{φ_1}/v_z versus v_*/v_z indicates sufficiently close agreement between the experimental data and the calculated curves based on Eq. (3) at various values of n.

The change of flow structures shown in Fig. 1 does not have much effect on the nature of the relationship between v_{φ_1}/v_z and v_*/v_z .

The drag coefficient of spherical particles may be assumed constant, which is valid on the interval $Re_{\varepsilon_0} = v_{\varepsilon_0} d/\nu = 10^3 - 10^5$.

In our investigation Re_{ε_0} varied between 10^3 and $5 \cdot 10^4$.

In the general case the coefficient c_w is variable and depends on Re_{ε_0}. Then expression (3) is written as

$$\frac{v_{\varphi_1}}{v_z} = \frac{1}{n}\sqrt{\frac{c_w^2(Re_*)}{(c_w')^2(Re_{\varepsilon_0})}\left(\frac{v_*}{v_z}\right)^4 - 1}. \quad (4)$$

The problem is one of considerable practical importance in connection with the removal of rock debris from bore holes and the operation of cyclone equipment.

REFERENCES

1. C. Gazley, Trans of the ASME, 80, no. 1, 1958.
2. S. I. Kosterin and Yu. P. Finat'ev, in: Heat Transfer and Cooling of Electrical Machines [in Russian], Moscow, 1963.
3. V. V. Mal'tsev, Vestnik elektropromyshlennosti, no. 6, 1959.
4. G. Taylor, Proc. Roy. Soc. (London) S. A., **157**, 1936.
5. L. A. Dorfman, Hydrodynamic Resistance and Heat Transfer of Rotating Bodies [in Russian], Fizmatgiz, Moscow, 1960.
6. V. V. Mal'tsev, in: Heat Transfer and Cooling of Electrical Machines [in Russian], Moscow, 1963.
7. B. P. Ustimenko and V. N. Zmeikov, Teplofizika vysokikh temperatur, no. 2, 1964.
8. S. I. Kosterin and Yu. P. Finat'ev, IFZh, no. 10, 1963.
9. F. A. Wattendorf, Proc. Roy. Soc., (A) **148**, 1935.
10. D. Z. Lozinskii, Izv. AN SSSR, OTN, no. 3, 1945.
11. C. E. Williams and G. H. Brucl, Trans ASME, 192, 111, 1951.

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